Hash Functions

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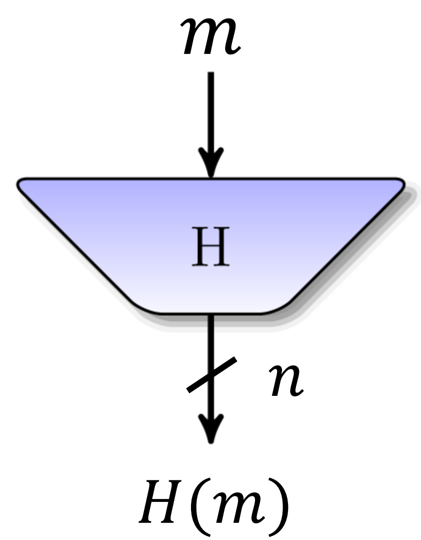
Hash function

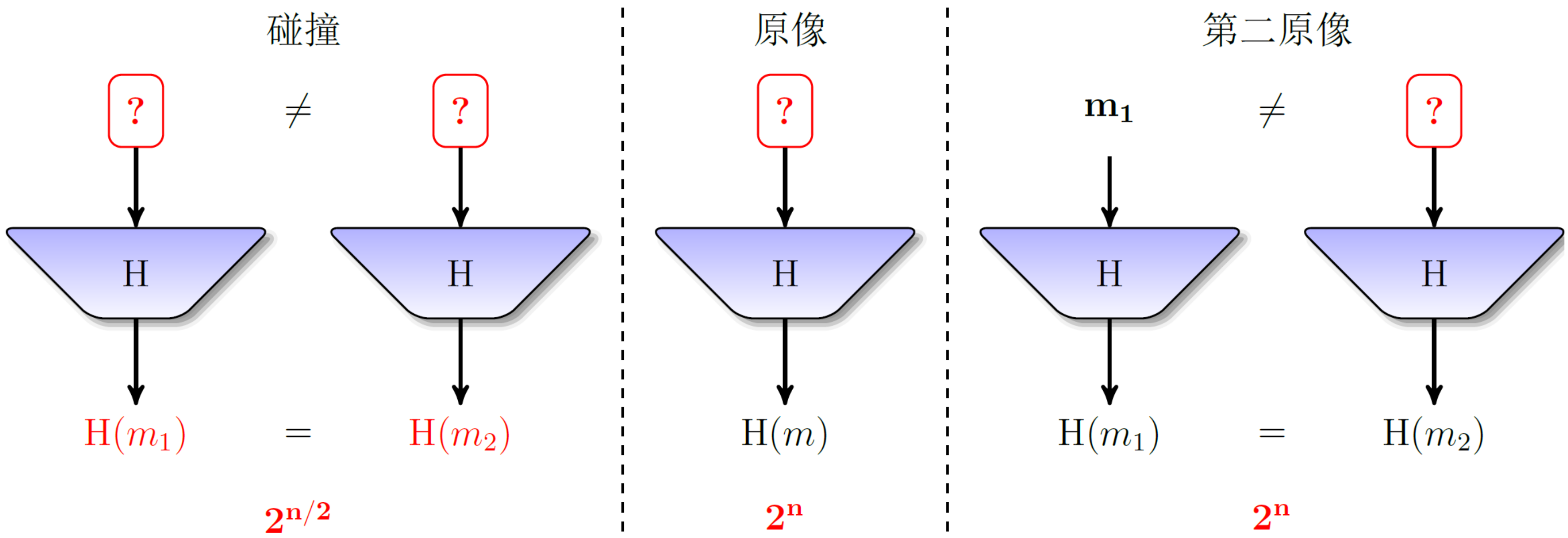
- Input: message of arbitrary size to data
- Output: fixed size, say n bits.



- H(m) is the fingerprint/digest/hash value of m

Cryptographic Hash Function

- Three properties of security



Cryptographic Hash Function

- Crypto hash function $h(x)$ must provide
  - **Compression** — output length is small
  - **Efficiency** — $h(x)$ easy to compute for any x
  - **Preimage resistance (One-way)** — given a value y it is infeasible to find an x such that $h(x) = y$
  - **Second-preimage resistance** — given x and $h(x)$, infeasible to find $y \neq x$ such that $h(y) = h(x)$
  - **Strong collision resistance** — infeasible to find any x and y, with $x \neq y$ such that $h(x) = h(y)$
- Many collisions exist, but cannot find any

Non-crypto Hash (1)

- Data $X = (X_0, X_1, X_2, \ldots, X_{n-1})$, each X_i is a byte
- Spse $\text{hash}(X) = X_0 + X_1 + X_2 + \ldots + X_{n-1}$
- Is this secure?
- Example: $X = (10101010, 00001111)$
- Hash is 10111001
- But so is hash of $Y = (00001111, 10101010)$
- Easy to find collisions, so **not** secure…

Non-crypto Hash (2)

- Data $X = (X_0, X_1, X_2, \ldots, X_{n-1})$
- Suppose hash is
  - $h(X) = nX_0 + (n-1)X_1 + (n-2)X_2 + \ldots + 1 \cdot X_{n-1}$
- Is this hash secure? At least

  $h(10101010, 00001111) \neq h(00001111, 10101010)$
- But hash of (00000001,00001111) is same as hash of (00000000,00010001)
- Not secure, but it is used in the (non-crypto) application

Non-crypto Hash (3)

- Cyclic Redundancy Check (CRC)
- Essentially, CRC is the remainder in a long division calculation
- Good for detecting burst **errors**
- Easy for Trudy to construct collisions
- CRC sometimes mistakenly used in crypto applications (WEP)

Popular Crypto Hashes

- **MD5** — invented by Rivest
  - 128-bit output
  - Note: MD5 collisions were found
- **SHA-1** — US NIST standard (similar to MD5)
  - 160-bit output
  - Deprecated recently
- **SHA-2** — US NIST standard (similar to SHA-1)
  - Most widely used nowadays
- **SHA-3** — US NIST standard
- **SM3** — Chinese standard

Popular Crypto Hashes

Year	Hash function	construction	NIST Standard (US)	NESSIE Standard (Europe)	CRYPTREC Standard (Japan)	国密标准
1990	MD4	MD				
1992	MD5	MD				
1995	SHA-1	MD	√		√	
1996	RIPEMD-160	MD			√	
2000	Whirlpool	MD		√		
2002	SHA-2	MD	√	√	√	
2010	SM3	MD				√
2015	SHA-3	Sponge	√		√	

Crypto Hash Motivation

- Digital signature

In 1976, Whitfield Diffie and Martin Hellman first described the notion of a digital signature scheme, although they only conjectured that such schemes existed based on functions that are trapdoor one-way permutations. Soon afterwards, Ronald Rivest, Adi Shamir, and Len Adleman invented the RSA algorithm, which could be used to produce primitive digital signatures (although only as a proof-of-concept – "plain" RSA signatures are not secure).

--- from Wikipedia

- In 1978, Rabin proposed the idea of signing the fingerprint of a document.

Public Key Notation

- **Sign** message M with Alice's **private key:** $[M]_{Alice}$
- **Encrypt** message M with Alice's **public key:** $\{M\}_{Alice}$
- Then

$$\{[M]_{Alice}\}_{Alice} = M$$

$$[\{M\}_{Alice}]_{Alice} = M$$

Crypto Hash Motivation: Digital Signatures

- Suppose Alice signs M
  - Alice sends M and $S = [M]_{Alice}$ to Bob
  - Bob verifies that $M = \{S\}_{Alice}$
- If M is big, $[M]_{Alice}$ is costly to compute
- Suppose instead, Alice signs $h(M)$, where $h(M)$ is much smaller than M
  - Alice sends M and $S = [h(M)]_{Alice}$ to Bob
  - Bob verifies that $h(M) = \{S\}_{Alice}$

Digital Signatures

- Digital signatures provide **integrity**
  - Like MAC
- Why?
- Alice sends M and $\mathrm{S} = [\mathrm{h(M)}]_{\mathrm{Alice}}$ to Bob
- If M changed to $\mathrm{M'}$ or S changed to $\mathrm{S'}$ (accident or intentional) Bob detects it:

  $\mathrm{h(M')} \neq \{\mathrm{S}\}_{\mathrm{Alice}}$, $\mathrm{h(M)} \neq \{\mathrm{S'}\}_{\mathrm{Alice}}$, $\mathrm{h(M')} \neq \{\mathrm{S'}\}_{\mathrm{Alice}}$

Non-repudiation

- Digital signature also provides for **non-repudiation**
- Alice sends M and $S = [h(M)]_{Alice}$ to Bob
- Alice cannot "repudiate" signature
  - Alice cannot claim she did not sign M
- Why does this work?
- Is the same true of MAC?

Non-non-repudiation

- Alice orders 100 shares of stock from Bob
- Alice computes **MAC** using symmetric key
- Stock drops, Alice claims she did not order
- Can Bob prove that Alice placed the order?
- **No!** Since Bob also knows symmetric key, he could have forged message
- **Problem:** Bob knows Alice placed the order, but he cannot prove it

Non-repudiation

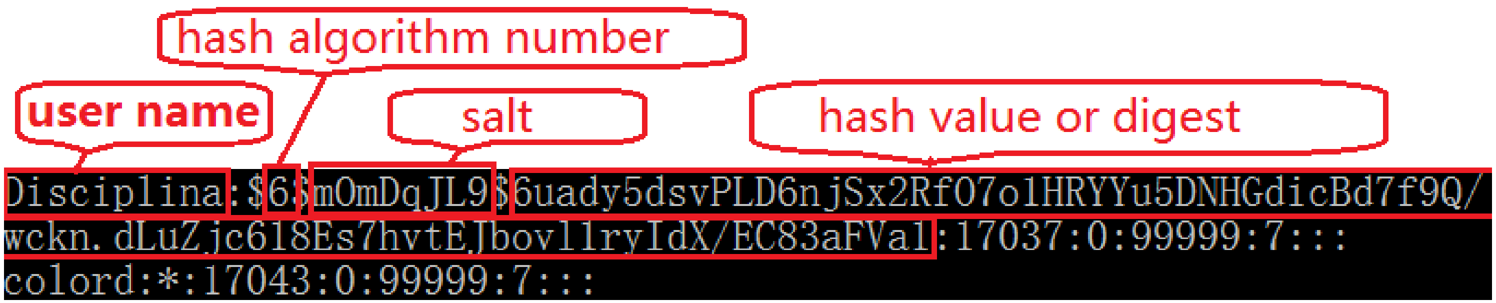
- Alice orders 100 shares of stock from Bob
- Alice **signs** order with her private key
- Stock drops, Alice claims she did not order
- Can Bob prove that Alice placed the order?
- **Yes!** Only someone with Alice's private key could have signed the order
- This assumes Alice's private key is not stolen (revocation problem)

Hashing and Signatures

- Alice signs $h(M)$, sends M and $S = [h(M)]_{Alice}$ to Bob and Bob verifies $h(M) = \{S\}_{Alice}$
- Security depends on public key system **and** hash function
- Suppose Trudy can find collision: $M' \neq M$ with $h(M') = h(M)$
- Then Trudy can replace M with M' and signature scheme is broken

Other applications

- Password protection



- Integrity verification

```
a55353d837cbf7bc006cf49eeff05ae5044e757498e30643a9199b9a25bc9a34 *ubuntu-18.04-desktop-amd64.iso
7a1c2966f82268c14560386fbc467d58c3fbd2793f3b1f657baee609b80d39a8 *ubuntu-18.04-live-server-amd64.iso
```

- Key generation
- Proof of work
  - Bit coin

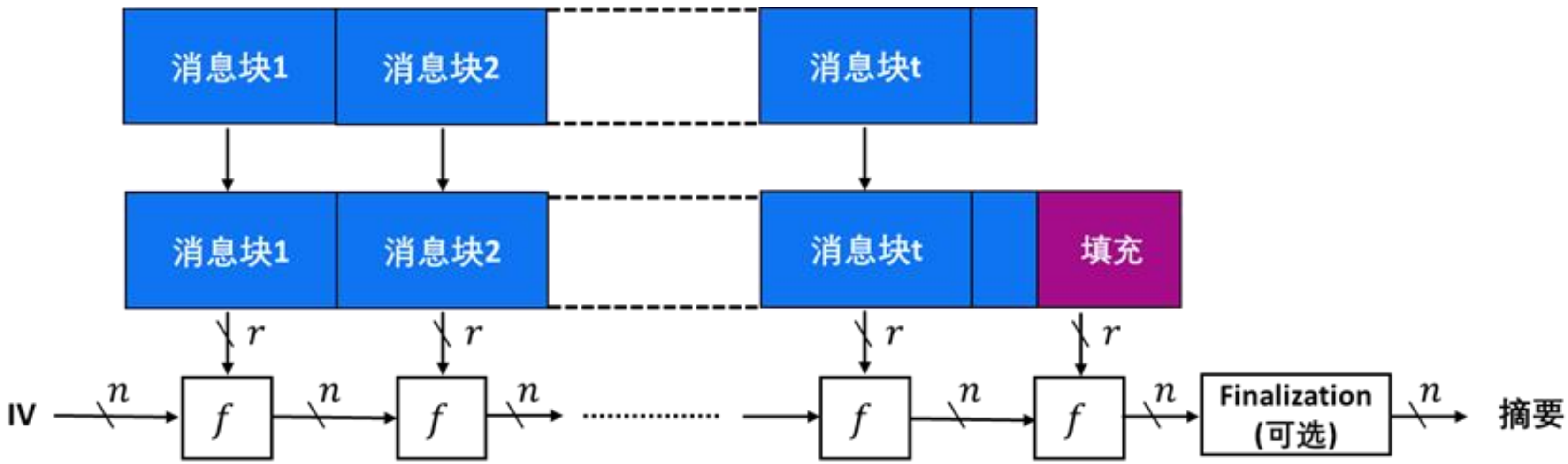
Crypto Hash Function Design

- Desired property: **avalanche effect**
  - Any change to input affects lots of output bits
- Crypto hash functions consist of some number of **rounds**
  - Analogous to block cipher in certain mode
- Want security and speed
  - Avalanche effect after few rounds
  - But simple rounds

Crypto Hash Function Design: MD construction

- Input data split into blocks
- Invoke a compression function iteratively
- **Compression function** applied to blocks
  - Current block and previous block output
  - Output for last block is the hash value
- For example
  - Block size is 512 bits
  - Compression function output is 128 bits

Crypto Hash Function Design: MD construction



This is known as Merkle-Damgård construction (1989).
E.g. n = 128, r = 512

Crypto Hash: Fun Facts for MD

- If msg is one 512-bit block: $h(M) = f(IV,M)$ where f and IV known to Trudy
- For 2 blocks:

  $h(M) = f(f(IV,M_0),M_1) = f(h(M_0),M_1)$
- In general $h(M) = f(h(M_0,M_1,\ldots,M_{n-2}),M_{n-1})$
  - If $h(M) = h(M')$ then $h(M,X) = h(M',X)$ for any X

Hashing and Birthdays

- The "birthday problem" arises in many crypto contexts
- We discuss it in hashing context
  - And "birthday attack" on digital signature
- Then Nostradamus attack
  - Learn how to predict the future!
  - Works against any hash that uses Merkle-Damgard construction

Pre-Birthday Problem

- Suppose t people in a room
- How large must t be before the probability someone has same birthday as me is $\geq 1/2$
  - Solve: $1/2 = 1 - (364/365)^t$ for t
  - Find $t = 253$

Birthday Problem

- How many people must be in a room before probability is $\geq 1/2$ that two or more have same birthday?
  - Suppose there are 365 days in a year.
  - Answer is 23.
- Why?

Birthday Problem

$$\frac{365}{365} \times \frac{365-1}{365} \times \cdots \times \frac{365-t+1}{365}$$

$$= 1 \times (1 - \frac{1}{365}) \times \cdots \times (1 - \frac{t-1}{365})$$

$$\approx 1 \times e^{-\frac{1}{365}} \times \cdots \times e^{-\frac{t-1}{365}} = e^{-\frac{t(t-1)}{2\times 365}}$$

Set $1 - e^{-\frac{t(t-1)}{2\times 365}} = 0.5$ and solve: **$t = 23$**

- Surprising? A paradox?
- No, it "should be" about $\sqrt{365}$ since compare **pairs** x and y

Birthday Problem – a generalized version

- Given a set with size N
- Choose t elements at random
- The probability p that at least one collision happens is $1 - e^{-\frac{t(t-1)}{2N}}$.
- Let $1 - e^{-\frac{t(t-1)}{2N}} = 0.5$, $\boldsymbol{t \approx 1.177\sqrt{N}}$.

Birthday attack on Hash functions

- Suppose a hash function *H* outputs *n*-bit digests, e.g., $n = 128$.
- Collision attack: find x_1; x_2 such that $H(x_1) = H(x_2)$
- Pick *t* inputs x_i , and compute $H(x_i)$
- Let $p = 0.5$, then $t=1.177\times 2^{128/2}$
- The brute-force attack of hash functions

Signature Birthday Attack

- Suppose hash output is n bits
- Trudy selects evil message E
  - Wants to get Alice's signature on E
- Trudy creates innocent message I
  - Alice willing to sign message I
- How can Trudy use birthday problem?

Signature Birthday Attack

- Trudy creates $2^{n/2}$ variants of I
  - All have same meaning as I
  - Trudy hashes each: $h(I_0), h(I_1), \ldots$
- Trudy creates $2^{n/2}$ variants of E
  - All have same meaning as E
  - Trudy hashes each: $h(E_0), h(E_1), \ldots$
- By birthday problem, $h(I_j) = h(E_k)$, some j,k

Signature Birthday Attack

- Alice signs innocent message I_j
- Then Trudy has $[h(I_j)]_{Alice}$
- But $[h(I_j)]_{Alice} = [h(E_k)]_{Alice}$
- Alice unwittingly "signed" evil E_k
- Attack relies only on birthday problem

Online Bid Example

- Suppose Alice, Bob, Charlie are bidders
- Alice plans to bid A, Bob B and Charlie C
  - They do not trust that bids will be secret
  - Nobody willing to submit their bid
- Solution?
  - Alice, Bob, Charlie submit **hashes** h(A),h(B),h(C)
  - All hashes received and posted online
  - Then bids A, B and C revealed
- Hashes do not reveal bids (one way)
- Cannot change bid after hash sent (collision)

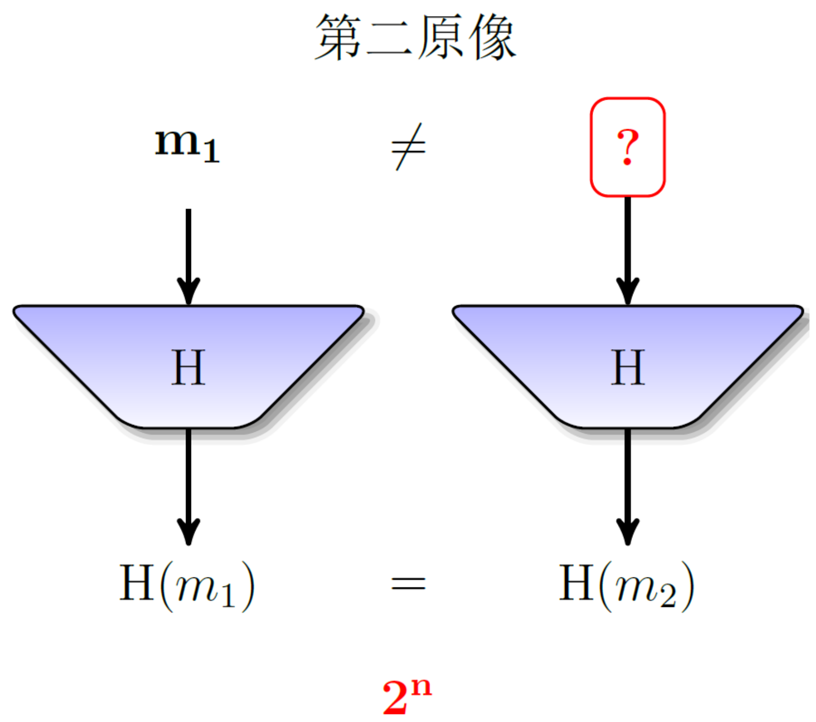
Online Bid

- This protocol is not secure!
- A forward search attack is possible
  - Bob computes h(A) for likely bids A
- How to prevent this?
- Alice computes h(A,R), R is random
  - Then Alice must reveal A and R
  - Trudy cannot try all A and R

Online Bid

- Spse $B = \$1000$ and Bob submits $h(B,R)$
- When revealed, $B = \$1000$ and $C = \$2000$
- Bob wants to change his bid: $B' = \$3$
- Bob computes $h(B',R')$ for different R' until he finds $h(B',R') = h(B,R)$
  - How much work?
  - Apparently, about 2^n hashes required

Second-preimage Attack



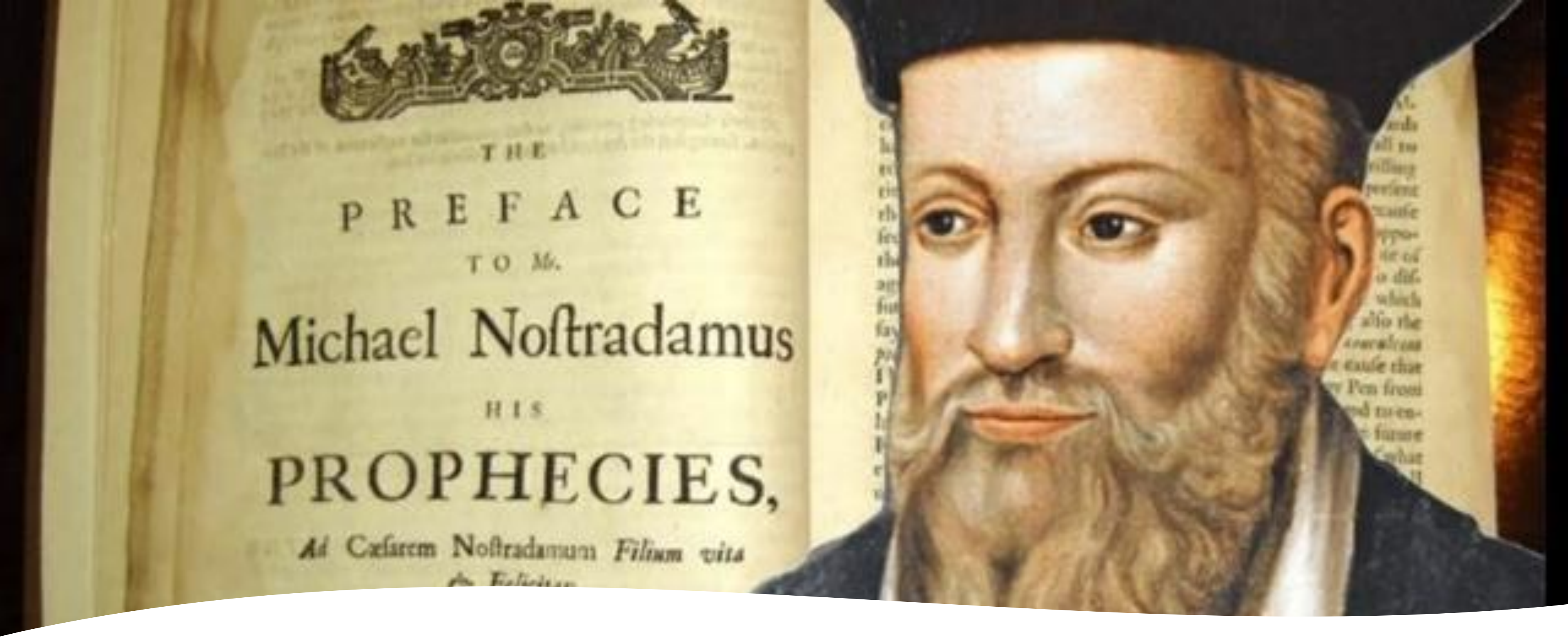
- Hash sometimes used to commit
  - For example, online bid example
- Attack on second preimages requires work of about 2^n hashes
- Collision attack is only about $2^{n/2}$
- Nostradamus attack solves second-preimage problem with only about $2^{n/2}$ hashes
  - For some cases, such as online bid example
  - Applicable to **any Merkle-Damgård hash**

Trudy Predicts Future?

- Trudy claims she can predict future
- Jan 1, 2021, she publishes y, claiming $y = h(x)$
  - Where x has final S&P 500 index for 2021 and other predictions for 2022 and beyond
- Jan 1, 2022, Trudy reveals x, with $y = h(x)$
  - And x has S&P 500 index for Dec. 31, 2021 along with other rambling predictions for 2022
- Does this prove Trudy can predict future?

Trudy Predicts Future?

- Trudy specifies y in advance
- Let P be S&P 500 for Dec 31, 2021
- Assuming Trudy cannot predict future, she must find S so that $y = h(P,S)$
- Trudy can hash 2^n different S
  - But, we assume this is too much work
  - Is there any shortcut?



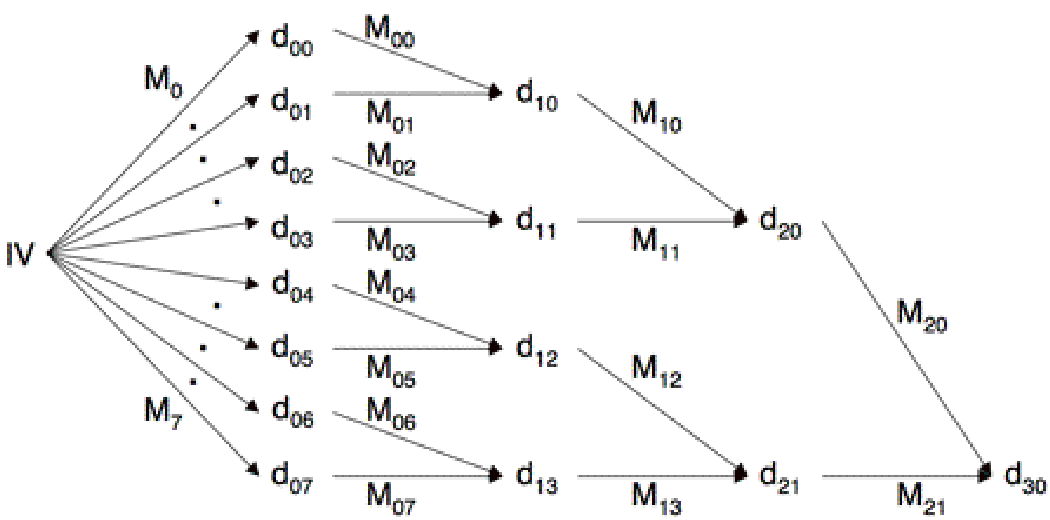
Nostradamus Attack

- Nostradamus (1503-1566) was a prophet
  - Some claim he predicted historical events
  - His predictive powers work best in retrospect
- Nostradamus attack
  - Trudy can predict the future
  - Convert 2^n second-preimage problem into about $2^{n/2}$ collision attack (essentially)
  - Applies to any Merkle-Damgård hash function

Nostradamus Attack

- Computing collisions: each $2 \cdot 2^{n/2}$ work
  - Comparing one set to another set
- Pre-compute collisions in clever way
- This determines y, the hash value
- When we specify prefix P, we can "herd" collisions into hash value y
  - Suffix S determined in this process

Diamond Structure



- Choose M_0 randomly
- Compute $d_{00} = f(IV, M_0)$
- And $M_1, \ldots, M_7$
- Then find M_{00}, M_{01} that give collision: $d_{10} = f(d_{00}, M_{00}) = f(d_{01}, M_{01})$
- Continue: $y = d_{30}$ is pre-determined hash

Nostradamus Attack

- Pre-computation
  - Compute diamond structure of "height" 2^k
  - Choose $y = d_{k0}$ as hash of prediction
- When "prediction" is known, Trudy will
  - Let P be "prediction"
  - Select S' at random, where (P,S') one block
  - Until she finds $f(IV,P,S')=d_{0j}$ for some j

Nostradamus Attack

- Once such S' is found, Trudy has result
  - Follow directed path from d_{0j} to d_{k0}
- In previous diamond structure example, suppose Trudy finds $f(IV,P,S') = d_{02}$
- Then $h(P,S',M_{02},M_{11},M_{20}) = d_{30} = y$
  - Recall that y is hash of Trudy's "prediction"
- Let $x = (P,S',M_{02},M_{11},M_{20})$
- And x is Trudy's "prediction": P is S&P 500 index, S',M_{02},M_{11},M_{20} are future predictions

Nostradamus Attack

- How much work?
- Assuming diamond structure is of height 2^k and hash output is n bits
- Primary: $2 \cdot 2^{n/2}(2^k - 1) \approx 2^{n/2+k+1}$
  - Can reduce this to $2^{n/2+k/2+1}$
- Secondary: 2^{n-k}

Nostradamus Attack

- To minimize work, set primary work equal to secondary work, solve for k
- We have $n/2 + k/2 + 1 = n - k$ which implies $k = (n - 4)/3$
- For MD4 or MD5, $n = 128$, so $k = 41$
- Diamond structure of height 2^{41}
- Total work is about 2^{87}

Nostradamus: Bottom Line

- Generic attack on any hash that uses Merkle-Damgard construction
- Not practical for 128-bit hash
  - Almost practical with small success prob
- Using hash to commit to something, is not quite as strong as it seems
- Weakness of MD construction

Summary

- Security requirements of crypto hash functions
- Applications:
  - Digital signature, integrity verification, …
- Brute-force attack: birthday attack
- MD construction
  - Weak second-preimage resistance